



# Radar observations of three comets and detection of echoes from one: P/Grigg-Skjellerup

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## Abstract

Radar echoes from the nucleus of the short-period comet Grigg-Skjellerup were obtained using the 12.6 cm wavelength radar of the Arecibo Observatory during the apparition of 20 May–2 June 1982. Dual circularly-polarized receiving channels were employed. The receiving mode orthogonally polarized to the transmission yielded an echo equal to nearly 10 times the standard deviation of the accompanying noise; no echo was detected in the polarization of the same sense as transmitted. The observations give a radar cross-section of  $0.50 \pm 0.13 \text{ km}^2$ , and an upper limit to the width of the (unresolved) echo spectrum of less than 1 Hz. These results are consistent with specular reflection from a solid nucleus having a radius of less than 0.4 km. The upper limit depends on the unknown rotation rate and scattering law. Two other comets, comets Austin and P/Churyumov-Gerasimenko have also been probed but have not returned any detectable echo, thus allowing one to put only an upper limit on the sizes of their nuclei. © 1999 Published by Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

As the source of all gas and dust observed during the appearance of a comet, the nucleus is of primary importance in the study of comets. Unfortunately, the presence of the coma usually prevents the direct observation of a nucleus using optical techniques, even when the comet is at large heliocentric distances.

However, radio waves can propagate almost unaffected through the coma. Thus, the main motivation for using radar to study comets lies in the fact that, unlike other groundbased astronomical techniques, radar offers the possibility to observe directly the nucleus of a comet and, simultaneously, to obtain information on its surface scattering properties, size, spin rate, and orbit.

The radar detectability of comets is governed by the radar equation:

$$\frac{S}{N} = \left[ \frac{P_t G^2 \lambda^{5/2} t_i^{1/2}}{4\pi k T_s} \right] \left[ \frac{\sigma}{32\pi \Delta^4 L \left( \frac{R^3}{\Omega_p} \right)^{1/2}} \right] \eta$$

where  $S/N$  is the signal-to-noise ratio,  $R$  the radius of the

(assumed) solid nucleus,  $\sigma$  the ratio of its radar cross-section to its geometric cross-section,  $\Omega_p$  the magnitude of the comet's apparent rotation vector projected on the celestial sphere in the direction of the line of sight from the radar,  $L$  the attenuation in the coma, and  $\Delta$  the geocentric distance of the comet. The effect on the radar detectability of the scattering law obeyed by the comet is characterized by  $\eta$ ; it is unity if the spectral density of the echo is constant from limb to limb and greater than unity otherwise. The parameters of the radar system are  $\lambda$  the wavelength of the transmitted signals,  $P_t$  the transmitted power,  $G$  the gain of the antenna relative to an isotropic radiator,  $T_s$  the effective system temperature, and  $t_i$  the integration time. These characteristics differ from one radar system to another. We must note that  $\sigma$  and  $\eta$  are also functions of  $\lambda$ .

In 1982, radar observations of three comets were attempted using the Arecibo Observatory's S-Band (2.38 GHz,  $\lambda = 12.6 \text{ cm}$ ) radar system: P/Grigg-Skjellerup, Austin and P/Churyumov-Gerasimenko. Echoes were detected only from P/Grigg-Skjellerup.

## 2. Observing procedure

All the observations were carried out using the 2.38 GHz radar system of the Arecibo Observatory. Each

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observing session, which lasted no more than about 2 h because of the limited sky coverage of the Arecibo antenna, was broken into cycles.

A cycle consisted of the transmission for a time approximately equal to the expected round-trip time of the radar signal, of a circularly polarized, nearly monochromatic signal. The frequency switching technique used has been described in an earlier paper (Kamoun et al., 1982a).

This transmission was followed by reception, for nearly the same duration, of the echo in the two senses of circular polarization, same as transmitted (SC), and opposite to that transmitted (OC). In view of the difficulty of detection of these small objects, and of the a priori uncertainty in the comets' ephemerides, only C–W transmissions were employed in order to limit the range of parameters over which to search for echoes. Ephemerides based on the comets' orbit, obtained from optical observations, were used to point the telescope and to adjust the receiver frequency.

### 3. Data taking and analysis

The processing of the received signals has been described in detail elsewhere (Kamoun, 1983). In brief it consists of sampling, Fourier analysis and correction for instrumental effects, and results in power spectra having a frequency resolution of about 1.0 Hz. Because of the low signal-to-noise ratio expected at each individual observing session, integration of the whole set of data obtained over the total observing period is usually necessary to detect an echo. The spectra presented below correspond to such integrations.

The primary objective of the data analysis is the estimation of the echo limb-to-limb bandwidth  $B$  and of the target radar cross-section  $\sigma$ . The echo was unresolved for P/Grigg-Skjellerup so that an investigation of the spectral shape of the echo, as described by Kamoun et al. (1982), could not be carried out, and only an upper limit on the limb-to-limb bandwidth could be obtained. Nonetheless, we were able to integrate the power over the echo bandwidth to obtain estimates of the radar cross-sections  $\sigma_{oc}$  and  $\sigma_{sc}$ .

For the other two comets, only upper limits on the radar cross-sections, and thus target size, have been obtained. These limits follow from upper limits on the echo bandwidth resulting from constraints imposed, on the one hand by the minimum spin period compatible with the gravitational stability of the nucleus ( $P_{min} \approx 4$  h) (Whipple (1982), Wyckoff (1982)), and on the other hand by reasonable assumptions on the scattering efficiency  $A$  of the nucleus' surface. The scattering efficiency is defined as the ratio of the estimated cross-section to the projected area of the target ( $A = \sigma/\pi R^2$  for a spherical target). Since radar studies of other bodies in

the solar system, from the inner planets to the Galilean satellites of Jupiter, have given values for  $A$  in the interval 0.04 to 1, it is very likely that the value for comet nuclei fall in this wide range. Using an upper limit on the radar cross-section, it is thus possible to also obtain an upper limit on the target's size.

Another estimate of a radar target's effective radius can be made using the Doppler broadening of the echo. These are linked by the relation:

$$R = \lambda BP / 8\pi \sin \theta$$

where  $\lambda$  is the radar wavelength, and  $\theta$  the angle between the nucleus' apparent spin axis and the radar line of sight. However, lack of knowledge of the direction of the nucleus' rotation axis precludes the use of this relation, except to obtain a lower bound on  $R$ . An estimate of the spin vector (Whipple and Sekanina, 1979) had been made for P/Encke, thus allowing the use of the estimated bandwidth of the 1980 radar echo to obtain a size estimate; however, until now no useful estimate has been obtained for any of the three comets presented here. A detailed discussion of the observations and results relative to these comets follows, each comet being treated separately and in chronological order of observation.

### 4. Comet P/Grigg-Skjellerup

With a 5.1 year orbital period, P/Grigg-Skjellerup has the second shortest orbital period among known comets. Although it has been observed optically at most of its perihelion passages since its discovery in 1902, very little data are available on its physical properties, in particular on the rotation of its nucleus. This comet was observed from Arecibo between May 20 and June 2, 1982, while the comet was at a geocentric distance of about 0.33 AU, about a week after it passed its perihelion. The system parameters and daily integration times for the observations are summarized in Table 1.

Table 1  
Summary of system parameters and total time of echo reception for Comet P/Grigg-Skjellerup

Date (1982)	$\Delta$ (AU)	$RTT$ (s)	$G$ (dB)	$P_t$ (kW)	$T_s$ (K)	$t_i$ (s)
May 20	0.351	350	71.5	400	43	2222
May 21	0.348	347	71.4	400	46	2904
May 23	0.343	342	71.4	400	46	2904
May 24	0.341	340	71.2	400	50	3043
May 26	0.338	337	71.1	400	53	3406
May 27	0.337	336	71.3	400	47	2620
May 29	0.335	334	71.0	400	50	1214
May 31	0.335	334	70.5	400	63	2620
June 2	0.336	335	70.2	400	70	2358

#### 4.1. Spectra

The spectra corresponding to the weighted sum of all the data obtained for the SC and OC polarizations are shown in Fig. 1 where power is given in units of standard deviations ( $1\sigma = \text{rms of noise power}$ ).

The echo signal from the nucleus of P/Grigg-Skjellerup is unresolved. All the power received appears to fall inside a single frequency resolution cell having a width of about 1 Hz, and lying at a frequency offset of 4 Hz from the expected a priori center frequency of the echo. Such an offset is within the estimated uncertainty for the Doppler prediction. Optical astrometry carried out before, during and after the radar observations have been used to obtain an improved a posteriori orbit.

Comparison of this new ephemeris with the a priori prediction ephemeris shows that the two ephemerides have a relative drift of the order of 1 Hz over the 14-day observing interval, so that some smearing of the summed echo spectrum should have occurred. The fact that the observed echo seems to be located at precisely the same offset for every day of observation raises some concern over its reality. To dispel this concern, two tests have been performed, one at the time of the observations, the other during the data analysis. During the period of observation, simulation tests were carried out on May 24 and June 6, 1982, using exactly the same set-up as for the normal comet observations, the only difference being that

no transmissions were made. The results of a 1-h integration on each of these two dates were negative, showing no sign of a spurious signal at the place where the echo was located under normal observing conditions.

The second test was carried out by processing the data in a manner different from that used normally. First we note that a spurious signal should not be expected to move across the received video bandwidth in synchronism with the four-frequency transmitted sequence as a real echo from the target should. Thus, we looked at the raw spectral data before shifting and superposing the contributions of the four spectral regions. The separate spectral regions corresponding to the transmission frequencies,  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_4$  (for the four days of observations May 20, 21, 23, 24 only, for practical reasons) have each been combined separately to yield four sums  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$ . The presence of the echo in each of the four plots at the same offset with respect to the ephemeris center frequency is evidence for the reality of the echo. Thus we are overwhelmingly convinced that the echo is real.

Now, in order to explain the apparent lack of drift of the echo, we note that a drift of the echo as high as 1 Hz over the observing interval would not be detected if the echo were intrinsically very narrow and if its mean position were centred on the spectral element where the received power is observed to fall. In particular, we can deduce that, at least on the first and last days of observation, the echo bandwidth was probably less than about 0.2 Hz. We also note that a scattering law with a sharply varying angular response could yield an effective bandwidth comparable to the one observed while permitting the total bandwidth of the echo to be substantially larger. In the following discussion, a value of 0.5 Hz for the upper limit of the total (limb-to-limb) bandwidth of the echo is assumed.

Such a narrow spectrum could result from some combination of:

- a very slow rotation of the nucleus
- a spin axis oriented nearly along the radar line of sight
- a very small nucleus.

We have seen above that the echo lies at a frequency offset of 4 Hz from its a priori center frequency. This information is useful for improving our knowledge of the comet's orbital motion, and may be stated as an inferred Doppler measurement of:

$$36969.2 \pm 0.5 \text{ Hz at } 22^{\text{h}}:17^{\text{m}} \text{ UT on May 26, 1982}$$

for a transmitted frequency of 2380 MHz.

#### 4.2. Radar cross-section

The echo power received in the OC sense, summed over all days, shows a signal-to-noise ratio of about 9.7 standard deviations. The radar cross-section equivalent

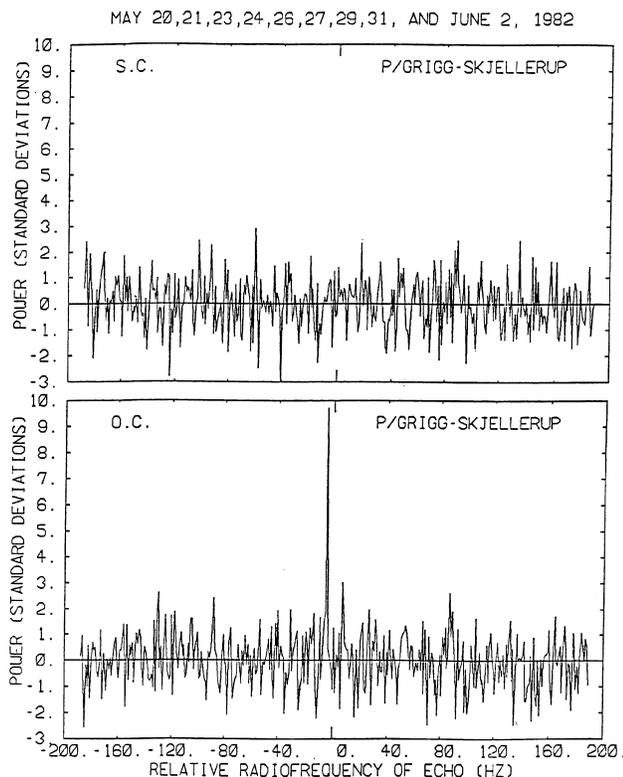


Fig. 1.

of the rms fluctuations in received noise power per frequency cell is about  $5.10^{-2} \text{ Km}^2$  for each polarization. The average OC radar cross-section is thus:

$$\sigma_{oc} = 0.50 \pm 0.13 \text{ km}^2$$

where the uncertainties come from consideration of both the noise fluctuations and the systematic errors in the calibration of system parameters. No echo was detected in the SC sense, so only an upper limit can be set on  $\sigma_{sc}$ . Assuming that the integrated power received in this sense is less than three standard deviations, we get:

$$\sigma_{sc} < 0.15 \text{ km}^2$$

for the radar cross-section in the SC sense.

By considering the lower bound on  $\sigma_{oc}$  and the upper bound on  $\sigma_{sc}$ , we can put a conservative upper limit on the circular polarization ratio  $\mu_c = \sigma_{sc}/\sigma_{oc}; \mu_c < 0.4$ .

Such an upper limit is not compatible with a pure solid ice surface for the nucleus, because such a surface would yield substantially more internal reflections.

#### 4.3. Size of the nucleus

The value estimated for  $\sigma_{oc}$  yields significant limits on the target size when used with the bounds given above for  $A$ . We get:

$$0.4 \text{ km} \lesssim R \lesssim 2.2 \text{ km}.$$

On the other hand, since an upper limit on  $B$  is available, an upper limit on the size of the nucleus could be obtained, were the rotation vector known. However, the estimates so far available (Larson, Sekanina, Whipple, private communications) are model-dependent and have very large error bars, so that in our context they do not yield useful results.

The limits on  $R$  ( $R > 0.4 \text{ km}$ ) and  $B$  ( $B < 0.5 \text{ Hz}$ ) do allow us to constrain the expression  $P/\sin\theta$  (Kamoun, 1983). We get:

$$P/\sin\theta > 2 \text{ days}.$$

A very detailed discussion of various geometrical, morphological and rotational configurations explaining the variation of nucleus radar cross-section is presented by Kamoun (1983).

### 5. Comet Austin

Comet Austin, discovered on June 19, 1982, was observed with the Arecibo S-Band radar system between August 8 and 12, 1982. Little time was available to schedule observations and prepare an accurate ephemeris, between the time of discovery and the most favorable time for a radar observation from Arecibo (August 10).

The radar system set up was identical to that used for the other comets, except on August 12 when, because of

Table 2

Summary of system parameters and total time of echo reception for Comet Austin

Date (1982)	$\Delta$ (AU)	$RTT$ (s)	$G$ (dB)	$P_t$ (kW)	$T_s$ (K)	$t_i$ (s)
August 8	0.329	328	71.4	150	46	2210
August 9	0.325	324	71.7	190	39	2242
August 10	0.325	324	71.5	400	42	3234
August 11	0.328	327	71.5	400	43	3250
August 12	0.335	334	71.5	400	41	2900

the ephemeris uncertainties and the failure to obtain an echo during the first days, an attempt to widen the search window was made by doubling the analysing bandwidth and the frequency resolution. The system parameters and daily integration times for these radar observations are shown in Table 2.

The ephemeris computed a posteriori from all the astrometric observations obtained during the appearance of the comet (June–November 1982) was substantially different from the ephemeris used during the radar observations. As a result, it appeared that there could have been a drift of as much as  $1 \text{ Hz h}^{-1}$  between the Doppler shifts of the echoes and the corresponding predicted Doppler shifts so that significant smearing of the signal could have occurred even in the span of a few hours of observations. Thus in the data analysis, we attempted to correct for this drift by combining the data using frequency shifts for each cycle's spectrum, derived from the a posteriori ephemeris.

The spectra corresponding to the weighted summation, with the shifting and combining of the data obtained from 8–11 August, were cross-correlated with different functions expected to approximate the spectral shape of the signal, in order to enhance the signal-to-noise ratio of any echo present. As can be seen on Fig. 2, this procedure did not reveal the presence of any echo. Hence, we can place only an upper limit on the nucleus' dimensions. The standard deviation of the noise fluctuation per spectral element (1 Hz spectral width) is  $0.06 \text{ km}^2$  for each received polarization. By considering the absolute upper limit on  $B$  compatible with  $P_{\min}$  ( $P_{\min} \simeq 4\text{h}$ ) and  $A_{\min}$ , we deduce a very conservative upper limit of about 2.7 km on the radius (Kamoun, 1983). Since larger values for  $P$ ,  $A$ , and a value for  $\theta$  lower than  $90^\circ$  are much more likely, we consider a value of 2 km as a very reasonable upper limit on the radius of the nucleus of comet Austin.

### 6. Comet P/Churyumov-Gerasimenko

Only three appearances of this short-period comet (6.6 years orbital period) had been recorded before the radar observations. In 1969, when discovered, in 1976, at an

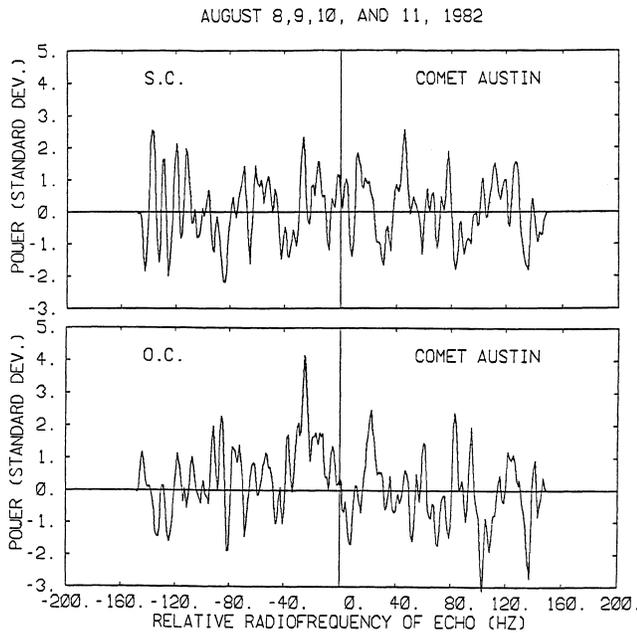


Fig. 2.

unfavorable return, and in 1982, when radar observations also took place between 8–15 November. The system parameters for these observations are given in Table 3.

Since the difference between the latest improved ephemerides and the one used during the observations changed by less than 0.2 Hz over the course of the observations, with an average offset of about 0.8 Hz, there should have been no significant smearing of the echo during the observing period. The spectrum corresponding to the weighted sum of all the data for the combination of all days was cross-correlated with various functions to attempt to obtain evidence for an echo.

Figure 3 shows the result of such a cross-correlation. Although there is a three standard deviation peak within

Table 3

Summary of system parameters and total time of echo reception for Comet Churyumov-Gerasimenko

Date (1982)	$\Delta$ (AU)	$RTT$ (s)	$G$ (dB)	$P_t$ (kW)	$T_s$ (K)	$t_i$ (s)
November 8	0.412	411	71.1	400	55	939
November 9	0.410	409	71.3	400	47	3130
November 10	0.408	407	71.3	400	50	3130
November 11	0.406	405	71.4	400	43	1305
November 12	0.405	404	71.1	400	51	2738
November 13	0.403	402	71.3	400	46	2809
November 14	0.401	400	71.2	400	51	3050
November 15	0.400	399	71.0	400	55	3314

$\Delta$  is the geocentric distance of the comet,  $RTT$  is the round trip time of the radar signal,  $G$  is the effective antenna gain,  $P_t$  is the average transmitted power,  $T_s$  represents the effective system temperature associated with the opposite-sense circular polarization,  $t_i$  is the total integration time of the echo.

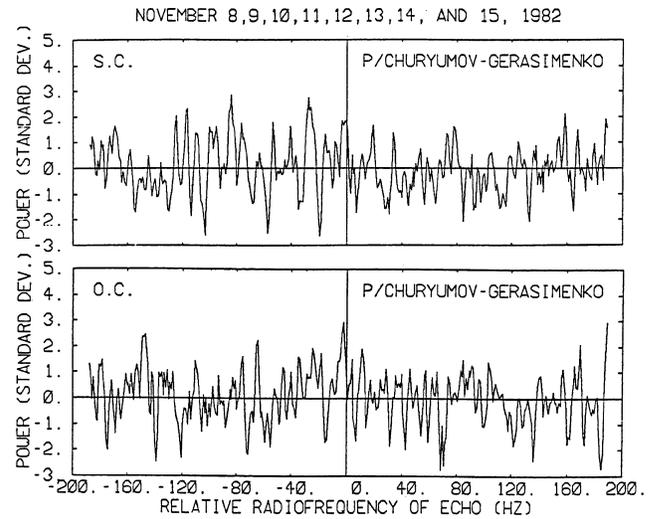


Fig. 3.

the ephemeris uncertainties, the evidence is too weak to claim a detection. We can however put an upper limit on the size of the nucleus using arguments similar to those presented above. We then obtain a strong upper limit of 3.7 km on the radius: a more reasonable upper limit is of the order of 3 km, given the fact that the values considered for  $P$ ,  $A$  and  $\Theta$  are very conservative.

## 7. Discussion

Besides providing the second direct detection of a comet nucleus (after the detection of comet Encke's nucleus in 1980), the radar observation of comet Grigg-Skjellerup's nucleus confirmed the presence of a solid nucleus and a small size consistent with the dusty snowball model. Moreover, these observations have allowed the estimation of the size of this nucleus and of upper limits on the size of two others.

The small size of the nucleus of comet Austin combined with its large production rates of water (Feldman, private communication) when compared to the corresponding values for Grigg-Skjellerup for instance (Weaver et al., 1981), with production rates higher by about two orders of magnitude for Austin, confirms that this difference in activity between comets is due more to the structure of the surface of the nuclei (extent of exposed volatiles) than to the actual sizes of the nuclei.

More observations by more powerful radar systems and/or at more favorable geometries, of these periodic comets, particularly the investigation of their spin vectors, will be necessary in order to take full advantage of the capabilities of radar. Such observations could also help in the interpretation of the variation of the radar cross-section of Grigg-Skjellerup with time, and for example in the placement of a lower upper limit on, or

the determination of, the radius of comet Churyumov-Gerasimenko.

Very few other radar observations of comets have been done since then (Campbell et al., 1983; Goldstein et al., 1983; Harmon et al., 1989, 1997) all leading to small sizes for the cometary nuclei.

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