Dynamics of the KW4 System

Simulation equations and model computation

The specific equations of motion solved for our simulations are stated in a compact form as (*1*):

$$
\begin{aligned}
m\ddot{r}_i &= U_{r_i} && (1)\\
\dot{H}^I_i &= -T^I_{ij}\epsilon_{jkl}T^I_{mk}U_{T^I_{ml}} && (2)\\
\Omega^I_i &= (I^I_{ji})^{-1}T^I_{kj}H^I_k && (3)\\
\dot{T}^I_{ij} &= T^I_{ik}\Omega^I_l\,\epsilon_{klj} && (4)\\
U(r,T^1,T^2) &= \mathcal{G}\int_{B^1}\int_{B^2}\frac{dm^1dm^2}{|r+T^1\rho^1-T^2\rho^2|} && (5)
\end{aligned}
$$

Here r is the relative position vector between the two centers of mass expressed in an inertial frame, the system mass parameter m equals the product of body masses divided by their sum, H^I is the inertial frame angular momentum vector of the Ith body, I^I is the inertia tensor of body I in its body-fixed frame, Ω^I is its angular velocity vector, T^I_{ij} is the attitude matrix of body I mapping its body-fixed frame to the inertial frame, ϵ_{ijk} is the skew-symmetric 3-tensor (with $\epsilon_{123} = 1$) that defines the cross product, $\mathcal{G}$ is the universal constant of gravitation, B^I signifies the mass distribution of the body with differential mass element dm^I, ρ^I is the location of that mass element in the Ith body frame, and U is the mutual gravitational potential between the bodies. Dots over a variable denote time derivatives, subscripts on all variables except U denote vector, matrix or tensor elements, and we assume the Einstein summation convention. A subscript on U denotes partial differentiation.