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RADAR ASTROMETRY OF NEAR-EARTH ASTEROIDS

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ABSTRACT

In an effort to assess the extent to which radar observations can improve the accuracy of near-Earth asteroid ephemerides, an uncertainty analysis has been conducted using four asteroids with different histories of optical and radar observations. We selected 1627 Ivar as a representative numbered asteroid with a long (56 yr) optical astrometric history and a very well-established orbit. Our other case studies involve three unnumbered asteroids (1986 DA, 1986 JK, and 1982 DB), whose optical astrometric histories are on the order of months and whose orbits are relatively poorly known. In each case study, we assumed that radar echoes obtained during the asteroid's most recent close approach to Earth yielded one or more time-delay (distance) and/or Doppler-frequency (radial-velocity) estimates. We then studied the sensitivity of the asteroid's predicted ephemeris uncertainties to the history (the schedule and accuracy) of the radar measurements as well as to the optical astrometric history. For any given data set of optical and radar observations and their associated errors, we calculated the angular (plane-of-sky) and Earth–asteroid distance uncertainties as functions of time through 2001. The radar data provided only a modest *absolute* improvement for the case when a long history of optical astrometric data exists (1627 Ivar), but rather dramatic reductions in the future ephemeris uncertainties of asteroids having only short optical-data histories. A few radar observations of a newly discovered near-Earth asteroid could easily mean the difference between successfully recovering the object during its next close approach to Earth and losing it entirely. Even for an asteroid with an extensive optical-data history, a few radar measurements can shrink the positional error ellipsoid by a factor of 2 for at least a decade.

I. INTRODUCTION

Near-Earth asteroids constitute an extraordinarily diverse population whose importance in the study of the solar system's early history is now firmly established. While there are fewer than one hundred known near-Earth asteroids, the entire population to absolute visual magnitude 18 (diameter about 1.6 ± 1.0 km) is estimated to be approximately 1300 (Shoemaker 1983). Typical dynamical lifetimes for these bodies are only 3×10^7 yr (Wetherill 1976), so the population would be quickly destroyed if it were not repopulated by the injection of new asteroids into Earth-crossing orbits. Sources of new near-Earth asteroids are thought to include quiescent comets as well as mainbelt asteroids. Some of these objects might be, or share, the parents of meteorites. The proximity and accessibility of many of these objects mark them as candidates for exploration during the next few decades and exploitation during the next century (O'Leary 1977; NEAR Report 1986). Finally, the roles of these objects in cratering the terrestrial planets, as agents of major biological extinctions in the past, and as potential threats to our civilization in the future have aroused a good deal of interest recently (Shoemaker 1983).

Given this variety of motivations to develop a reliable census of the near-Earth asteroid population and to ascertain the nature of a substantial sample of individual objects, several search programs designed to discover them have been initiated (Helin and Shoemaker 1979; Gehrels *et al.* 1986). Additionally, a considerable effort is devoted to the astrometric observations and computations required to maintain ephemeris reliability for known near-Earth and mainbelt asteroids (Marsden 1979). In fact, the estimation of orbital elements and the generation of prediction ephemerides are a fundamental prerequisite to each step in the study of asteroids. Especially accurate ephemerides are needed for stellar occultation timing and for radar investigations. The most stringent requirements will arise for asteroids targeted for spacecraft flyby and rendezvous missions (Marsden 1971; Friedlander *et al.* 1979).

Reliable prediction ephemerides tend to be more difficult to develop for near-Earth asteroids than for mainbelt asteroids, for several reasons. They are favorably placed for optical observations less frequently and for shorter durations than are mainbelt asteroids. Because of their relatively fast apparent motions, near-Earth asteroids can be more difficult to detect on photographic plates than are equally bright mainbelt objects. Moreover, it is often difficult to recover them during post-discovery returns because their ephemerides are based upon observations taken over relatively short time periods and their motions are subjected to strong perturbations by the inner planets.

Radar observations have contributed to the accuracy of planetary ephemerides since the early 1960s, when radar ranging to Venus permitted improvements in our knowledge of the astronomical unit by several orders of magnitude (Pettengill and Shapiro 1965) and radar refinements of the ephemerides of Mercury, Venus, Earth, and Mars ensured successful guidance of the first interplanetary space probes (Ash *et al.* 1967; Newhall *et al.* 1983). Since then, radar data have helped maintain the accuracy of planetary ephemerides and have permitted dynamical studies ranging from tests of general relativity (Shapiro *et al.* 1968a, b; Canuto *et al.* 1984) to dynamical investigations of Jupiter's Galilean satellites (Chandler 1979).

Radar observations of asteroids have increased dramatically since 1980, and echoes from 17 near-Earth asteroids and 29 mainbelt asteroids have furnished a wealth of new information about asteroid characteristics (Ostro 1985; Ostro *et al.* 1985, 1986a,b). However, we are unaware of any

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thorough assessment of how radar measurements can improve asteroid ephemerides.

Here we report the results of numerical investigations into the effects of radar observations on asteroid ephemeris uncertainties. Of special interest is the question of how uncertainties in predicted plane-of-sky asteroid positions vary as functions of time for several years after a given series of optical astrometric measurements, and how radar measurements can decrease those uncertainties. We have performed case studies of four near-Earth asteroids with very different observational histories. Our uncertainty analyses were designed to sample a huge parameter space briefly, but in a sensible and informative manner. They are far from exhaustive, but our results do offer a representative picture of the potential contributions of "radar astrometry" to various stages of asteroid exploration.

II. UNCERTAINTY-ANALYSIS TECHNIQUE

Our uncertainty-analysis technique is best described using the formalism and notation of least-squares estimation theory (see Lawson and Hanson 1974). Let $\mathbf{x}_0$ denote the asteroid's state at a certain epoch (six components of position and velocity, measured in an inertial frame), and let $\mathbf{z}$ denote a vector of m observations of the asteroid made at various times. Based on the measurements $\mathbf{z}$ and an *a priori* estimate of the state $\tilde{\mathbf{x}}_0$, the least-squares method can be used to compute an improved estimate $\hat{\mathbf{x}}_0$ of the state. The measurement residuals $\delta\mathbf{z}$, defined in the sense of "observed value minus computed value," are assumed to be related to variations $\delta\mathbf{x}_0$ in the epoch state by the linear equation

$$\delta\mathbf{z} = \mathbf{A}\delta\mathbf{x}_0 + \boldsymbol{\nu}, \tag{1}$$

where $\mathbf{A}$ is an $m\times 6$ matrix of partial derivatives, in which element A_{ij} is the partial derivative of measurement z_i with respect to state variable x_j, and $\boldsymbol{\nu}$ is an m vector of unknown measurement errors. The value of $\mathbf{x}_0$ that minimizes the weighted sum of squares of the residuals is the least-squares state estimate $\hat{\mathbf{x}}_0$, given by

$$\hat{\mathbf{x}}_0 = \tilde{\mathbf{x}}_0 + (\mathbf{A}^{\mathrm{T}}\mathbf{W}\mathbf{A})^{-1}\,\mathbf{A}^{\mathrm{T}}\mathbf{W}\,\delta\mathbf{z}, \tag{2}$$

where $\mathbf{W}$ is a weighting matrix comprised of the inverses of the measurement variances σ_k^2 along the main diagonal. We assume that the measurement errors are uncorrelated, so that the off-diagonal elements of $\mathbf{W}$ are zero. The covariance matrix for errors in the epoch state estimate, which is a measure of the uncertainty in the estimate, is given by

$$\mathbf{P} = (\mathbf{A}^{\mathrm{T}}\mathbf{W}\mathbf{A})^{-1}. \tag{3}$$

We refer to $\mathbf{A}^{\mathrm{T}}\mathbf{W}\mathbf{A}$ as the information matrix, and denote it as $\boldsymbol{\Lambda}$.

In an uncertainty analysis, only the error covariance $\mathbf{P}$ is of interest. This contrasts with full state estimation, in which calculating the state estimate $\hat{\mathbf{x}}_0$ is the primary objective. Note that the error covariance can be computed directly from the information matrix, without computing the estimate. Furthermore, the error covariance depends only on the measurement noise statistics $\mathbf{W}$ and the observation partial derivatives $\mathbf{A}$, so that it is not necessary to have the measurements themselves in order to perform the analysis.

In order to study how the ephemeris uncertainties vary with time, it is necessary to simulate the time evolution of the error-covariance matrix. The customary method for solving this problem is to use a sequential estimation technique such as the extended Kalman filter (Jazwinski 1970) instead of a "batch" estimation technique such as that described above. Rather than taking the conventional approach, however, we chose to use a variation of batch estimation in which the error-covariance matrix at any given time of interest t is obtained by first computing the epoch error covariance based on only the subset of observations made up to that time, and then mapping the result to time t. The computation is repeated for each time of interest. This pseudosequential technique has been observed to be numerically more stable than sequential estimation methods (Chodas 1986).

Consider the k th observation z_k made at time t_k, and suppose that we wish to compute the ephemeris uncertainties immediately after this observation has been made. First, the observation is used to compute a new epoch error covariance. The required partial derivative $\mathbf{a}_k^{\mathrm{T}}$ of the observation with respect to the epoch state $\mathbf{x}_0$ is computed as follows:

$$\mathbf{a}_k^{\mathrm{T}} \triangleq \left[\frac{\partial z_k}{\partial e_1}\cdots\frac{\partial z_k}{\partial e_6}\right] = \left[\frac{\partial z_k}{\partial x_1}\cdots\frac{\partial z_k}{\partial x_6}\right]\begin{bmatrix}\frac{\partial x_1}{\partial e_1} & \cdots & \frac{\partial x_1}{\partial e_6}\\ \vdots & & \vdots\\ \frac{\partial x_6}{\partial e_1} & \cdots & \frac{\partial x_6}{\partial e_6}\end{bmatrix}, \tag{4}$$

where $e_1,\ldots,e_6$ are the components of the epoch state, and $x_1,\ldots,x_6$ are the components of the state at the time of the observation. The first factor in the above product depends on the type of observation being made, and is computed analytically. The second factor is the state transition matrix $\boldsymbol{\Phi}(t_k)$, which indicates how errors in the state at epoch map into errors at time t_k. This matrix is computed along with the state components $x_1,\ldots,x_6$ via numerical integration. Note that $\mathbf{a}_k^{\mathrm{T}}$ is the k th row of $\mathbf{A}$.

The information matrix $\boldsymbol{\Lambda}_k$ resulting from the processing of the first k observations can now be computed using the first k rows of $\mathbf{A}$. It is not necessary, however, to completely recompute the information matrix after each observation, since the following recursive relation can be used:

$$\boldsymbol{\Lambda}_k = \boldsymbol{\Lambda}_{k-1} + \mathbf{a}_k\mathbf{a}_k^{\mathrm{T}}\sigma_k^{-2}, \tag{5}$$

where σ_k is the standard deviation of the measurement noise for observation z_k. It is now simple to compute the epoch error-covariance matrix $\mathbf{P}_k$, by inverting $\boldsymbol{\Lambda}_k$, as in Eq. (3). Note that the *a priori* information matrix $\mathbf{A}_0$ is zero, since nothing is known about the state before the first observation.

Finally, the epoch error-covariance matrix $\mathbf{P}_k$ is mapped to various times of interest t via the relation

$$\mathbf{C}(t) = \boldsymbol{\Phi}(t)\mathbf{P}_k\,\boldsymbol{\Phi}(t)^{\mathrm{T}}, \quad t_k \leqslant t < t_{k+1}, \tag{6}$$

where $\boldsymbol{\Phi}(t)$ is the state transition matrix, defined earlier. The mapped covariance matrix $\mathbf{C}(t)$ is then transformed from the inertial coordinate system to either the *R-T-N* or the plane-of-sky coordinate system. The *R-T-N* coordinate system is defined by orthonormal basis vectors along the radial vector from the Sun, the transverse direction in the orbital plane, and the direction normal to the orbital plane. The plane-of-sky system is defined with one basis vector aligned along the Earth–asteroid direction, and the two others in the plane of the sky as viewed from the Earth. The *R-T-N* position uncertainties quoted in the next section are the square roots of the first three diagonal elements of the transformed error-covariance matrix $\mathbf{C}_{RTN}$. Note that the top left 3×3 submatrix of the error-covariance matrix represents a constant-probability ellipsoid for the uncertainty in the asteroid's position. The square roots of the first three diagonal elements define a box which just encloses the error ellipsoid. In the next section, "plane-of-sky uncertainty" denotes the

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apparent angular size of the semimajor axis of the error ellipse created by projecting the error ellipsoid onto the plane of the sky.

Although we have described our uncertainty analysis in terms of the least-squares method, the technique we actually used for this study is not least squares, but rather an equivalent method formulated in terms of the square root of the information matrix (Bierman 1977). This alternate method is numerically superior to least squares, since it avoids the explicit computation of the information matrix, thereby reducing the accumulation of error due to numerical roundoff.

The three types of observations used in this study are optical angle (i.e., a right ascension and a declination), radar time delay, and radar Doppler frequency. We assume a time-delay datum to be a measurement of the distance or "range" between the centers of mass of the Earth and the asteroid, and a Doppler datum to be a measurement of the time derivative of that distance. (This simplification is appropriate for our present purposes, but glosses over certain fundamental theoretical and experimental considerations; see, for example, Ash *et al.* 1967). The potential radar Doppler and range accuracies that can be achieved for spherical bodies having rough surfaces have been studied by Jurgens and Bender (1977). For our current study, realistic standard deviations for the Doppler and range measurements were chosen with the object's dimensions and geocentric distance in mind. Large upper bounds upon these values were selected to let us explore the sensitivity of the uncertainties to widely varying radar accuracies.

The standard deviation chosen for the optical measurement noise was either 1″ or 1.5″ in each coordinate; these values correspond to the root-mean-square residuals obtained in the recent orbit determinations using only optical data. The times of simulated optical data were chosen such that the Earth–asteroid distances were small enough and the solar elongation angles were large enough to allow optical observations to be taken.

We assume throughout this study that measurement noise is the dominant error source in the asteroid-orbit improvement process. All other errors, such as those in the observatory locations and planetary positions and masses, have been neglected. As a result, the absolute uncertainties given in the next section are somewhat optimistic, although the relative results are valid.

III. RESULTS

Table I outlines the rationale for our four case studies, which together comprise a variety of possible optical and radar data histories for near-Earth asteroids, and which are described in detail in this section.

Our first object of study, asteroid 1627 Ivar, has a long history of astrometric observations dating back to 1929 and was observed with the Arecibo radar during its approach to within 0.2 AU of the Earth in July 1985. With its extensive optical astrometric history and 1985 radar observations during a close Earth approach, 1627 Ivar represents one extreme of our sample. As an example of an object with a short optical astrometric history and radar observations made at relatively small geocentric distances, we chose asteroid 1986 DA. We selected asteroid 1986 JK as an object with radar observations made during an extremely close Earth approach but lacking extensive astrometric observations. Finally, we examined the currently lost asteroid 1982 DB in an effort to see how hypothetical radar observations, in early 1982, might have improved the chances of recovering this object.

Our strategy in each study was to investigate the positional uncertainties for each object during future returns when the ephemerides are generated (i) using only optical data obtained through the most recent apparition, (ii) using radar data as well as the existing optical data, and (iii) by augmenting the "nominal" data sets in (i) and/or (ii) with hypothetical (simulated) future optical and/or radar data. For each of our cases, Table II gives the initial conditions at epoch ($\mathbf{x}_0$) used to generate a reference ephemeris. Table II also gives the actual and simulated observation intervals, as well as the assumed error sources and their standard errors (i.e., the "noise") in the optical and radar measurements. The intervals spanned by our assumed radar noise levels overlap values realized in recent radar observations of near-Earth asteroids with the Arecibo and Goldstone instruments, but have been widened to reveal the sensitivity of ephemeris uncertainties to the accuracy of Doppler and range measurements. The observations used for a particular asteroid in our study do not necessarily represent the most complete data set available.

In a typical case study, we augmented optical data with radar by first including a single Doppler datum on a certain

TABLE I. Rationale for selecting asteroid case studies.

Case study	Asteroid	Optical astrometric history	Radar astrometric history
1	1627 Ivar	Excellent	Range and Doppler observations in 1985 at 0.2 AU from Earth
2	1986 DA	Good	Range and Doppler observations 2 months after discovery and at 0.2 AU from Earth.
3	1986 JK	Fair	Doppler observations 3 weeks after discovery and at 0.03 AU from Earth.
4	1982 DB	Poor	None attempted. Post-discovery detection not possible with available instrumentation.

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TABLE II. Error-analysis assumptions.

Asteroid	1627 Ivar	1986 DA	1986 JK	1982 DB
Reference for Orbit Employed:	Yeomans(1986a) MPC 10752	Urata(1986) IAU Circ. 4186	Yeomans(1986b) IAU Circ. 4220	Yeomans Unpublished
Epoch (E.T.)	6/19.0/86	6/19.0/86	6/19.0/86	1/11.0/82
T (E.T.)	7/14.1782/85	4/7.6197/86	7/1.8000/86	1/15.5838/82
q (AU)	1.1240960	1.166000	0.896892	0.953219
e	0.3966274	0.586325	0.684269	0.361080
ω (deg., 1950)	167.33513	126.7066	232.4143	157.9308
Ω (deg., 1950)	132.67498	64.5487	62.2412	314.2073
i (deg., 1950)	8.44331	4.3009	2.1494	1.4136
Employed Optical Observations:				
Number	242	33	29	19
Dates	9/25/29 - 7/18/85	2/5/86 - 3/16/86	5/4/86 - 5/27/86	2/28/82 - 5/14/82
Simulated Optical Observations:				
Number	40	18	5	10
Dates	1/29/90 - 12/15/90 9/30/93 - 1/8/94 1/23/95 - 6/12/95 9/24/98 - 3/3/99	4/2/86 - 4/20/86 7/3/91 - 8/12/91 11/4/94 - 12/14/94 8/5/96 - 9/14/96 11/18/99 - 12/28/99	9/7/86 - 9/17/86 10/10/95 - 10/30/95	8/27/90 - 10/26/90 9/25/92 - 1/3/93
Simulated Radar Observations:				
Date	7/5/85	4/10/86	5/28/86	3/7/82
Geocentric dist.	0.21 AU	0.21 AU	0.028 AU	0.13 AU
Date	7/9/85	4/20/86	5/30/86	3/14/82
Geocentric dist.	0.20 AU	0.21 AU	0.031 AU	0.15 AU
Error Sources:				
Optical data noise (arc sec)	1.0	1.0	1.0	1.5
Doppler data noise (cm/s)	10	1, 30	2, 20	2, 20
Range data noise (km)	1.0	0.5, 10.0	0.2, 2.0	0.3, 30.0

Notes to TABLE II

The orbital elements given in the table are appropriate for the given epoch of osculation. The elements listed are the perihelion passage time (T) in ephemeris time, perihelion distance (q) in AU, eccentricity (e), argument of perihelion (ω), longitude of the ascending node (Ω), and the inclination (i). The angular elements are referred to the ecliptic plane and the equinox of 1950.

date, then including a second Doppler datum on a date a few days later, and then including a range datum on one or both of those dates. (In general, the waveforms used in planetary radar ranging also yield a Doppler measurement.) Note from Table II that, compared to the optical data sets, the radar data sets are very small and span short time intervals. We also examined the sensitivity of the ephemeris uncertainties to *when* radar observations were taken and found that the optimum time for making Doppler observations is when the Earth–asteroid radial velocity (absolute Doppler) is largest. Conversely, the worst time for making Doppler observations (from an ephemeris-improvement viewpoint) is at the time of the Earth close approach when the radial velocity is zero.

In addition, for a given number of Doppler observations, it is better to take them all on optimal days rather than to spread them out over less favorable days (i.e., days when the radial velocity is relatively low). When range observations are added to the Doppler measurements, we could find no general conclusions concerning the optimal times for the radar observations. In an effort to keep the scope of our calculations manageable, we have not treated the radar measurement *times* as free parameters in the following case studies.

a) Case Study #1: 1627 Ivar

This object has been observed during 12 apparitions since its 25 September 1929 discovery at Johannesburg by E. Hertzsprung, and the frequency of its close approaches to the Earth ensures that it will be well observed in the future.

Table III summarizes our results for asteroid 1627 Ivar, and also allows us to calibrate certain aspects of our method-

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TABLE III. Ephemeris uncertainties for 1627 Ivar for the dates 24 June 1985, 9 May 1990, and 3 May 1995. When radar observations are included in the data set, both Doppler and range observations were assumed to have been made on 5 and 9 July 1985.

Dataset	Optical astrometric data (no.)	Plane of sky uncertainty (arc-sec) June 1985 Actual	June 1985 Pred.	May 1990 Pred.	Range uncertainty (km) June 1985 Pred.	May 1990 Pred.	May 1995 Pred.
1962-1984 optical	83	4.4	3.3	1.9	500	439	600
1929-1985 optical	242	0.6	0.39	0.20	52	30	40
1929-1985 optical + all radar	242	-	(0.39)	0.13	(52)	12	16
1929-1995 optical	271	-	0.39	0.19	52	30	36
1929-1995 optical + all radar	271	-	(0.39)	0.13	(52)	11	15

Note to Table III
The entries in parentheses do not reflect the radar data because the reference time (June, 1985) is prior to the assumed date of the radar observations (July, 1985).

ology. Each row in Table III corresponds to a particular set of data used in our simulations, and the last six columns give plane-of-sky and Earth–asteroid distance uncertainties for the dates 24 June 1985, 9 May 1990, and 3 May 1995. The geocentric distances were 0.2, 0.5, and 0.8 AU, respectively, on these three dates (see Fig. 1), which were chosen to be representative of times when the asteroid could be observed easily with optical telescopes.

For the June 1985 date, Table III compares the actual rms residual for the optical astrometric observations made during that month to the rms uncertainty that our analysis predicted for the same period. As expected, the size of the rms residual decreases (from 4.4″ to 0.6″) as the astrometric data arc was expanded from 1962–1984 to 1929–1985. A similar trend is evident in the values of the plane-of-sky uncertainties predicted by our covariance analysis (a decrease from 3.3″ to 0.39″), but the corresponding actual rms residuals are several tens of percent larger than our predicted uncertainties. This calibration experiment and others like it indicate that our covariance calculations yield plane-of-sky uncertainty predictions whose relative values are very reliable but whose absolute values are likely to be optimistic by several tens of percent.

Ivar's extensive astrometric history allowed some experimentation to determine the sensitivity of future ephemeris uncertainties to optical astrometric data intervals. We came to the expected conclusion that extending the data interval substantially improves the uncertainties of future ephemer-

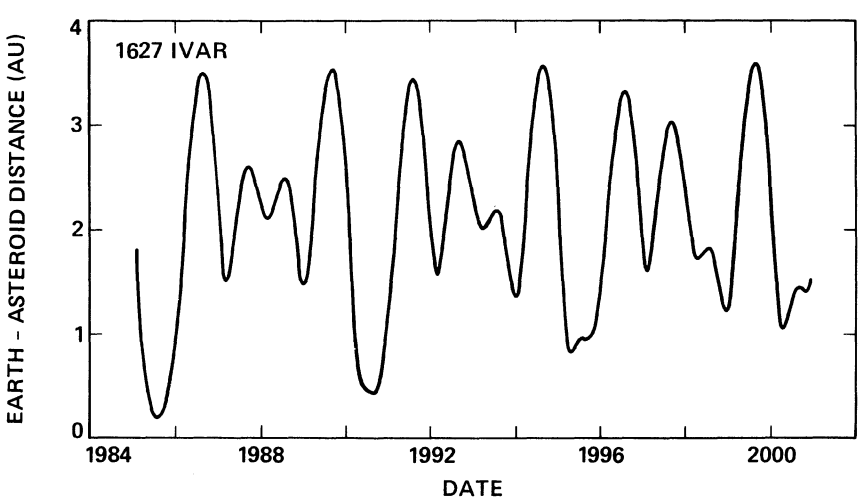

FIG. 1. 1627 Ivar: Asteroid's geocentric distance (AU) versus calendar date.

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ides. For example, the plane-of-sky angular ephemeris uncertainties in June 1985 and May 1990 were 3.3″ and 1.9″, respectively, when 1962–1984 optical, astrometric data were used. The inclusion of data over the longer 1929–1985 interval reduced the 3.3″ uncertainty to 0.39″ in June 1985 and 0.20″ in May 1990. (For a given linear position uncertainty at the asteroid, the angular uncertainty increases with decreasing geocentric distance. Hence the angular uncertainty is larger in 1985 than it is in 1990 because the asteroid comes a factor of 2 closer in 1985.)

Radar observations of Ivar, conducted at Arecibo during 5–10 July 1985 by Ostro *et al.* (1985), have yielded echoes resolved in Doppler frequency and, for one date, in time delay. These data will provide velocity and distance estimates with uncertainties of about 10 cm/s and 1 km. For our analysis, we assumed that two pairs of radar observations (Doppler and range) were made on 5.0 July and 9.0 July 1985. When these simulated radar data are included in our covariance analysis, the plane-of-sky uncertainty in the May 1990 ephemeris is reduced from 0.20″ to 0.13″. Hence, augmenting 242 optical-angle measurements with just two radar range/Doppler measurements achieves a 65% reduction in the plane-of-sky ephemeris uncertainty five years after the radar observations were taken. Although the addition of radar observations allows only a small *absolute* improvement in Ivar's long-term ephemeris, this improvement might prove useful in refining future predictions of stellar-occultation track locations.

The last three columns of Table III give our calculated uncertainties in the Earth–Ivar distance for three apparitions and five different astrometric data sets. Radar astrometry reduces range uncertainties to approximately 40% of the values corresponding to optical-only data sets. For both the plane-of-sky and range ephemerides the improvement due to radar astrometry lasts for at least a decade past the epoch of the radar observations.

Ivar is one of several objects under consideration for a spacecraft-rendezvous mission during the 1990s (NEAR Report 1986). One of the most practical implications of our calculations is that, even for an extensively observed asteroid, a few radar measurements can shrink the positional error ellipsoid by a factor of 2 for many years—perhaps resulting in more accurate targeting of a future flyby or rendezvous spacecraft arriving more than a decade after the radar observations were made.

b) Case Study #2: 1986 DA

M. Kizawa (1986) discovered this asteroid in Shizuoka, Japan, on 16 February 1986. Pre-discovery observations are available from February 5, and we have included actual observations through March 16 and simulated observations through April 20 in our analysis.

Figure 2 shows the Earth–asteroid distance as a function of time, and Fig. 3 shows the 1986–2001 progression of ephemeris uncertainties in the asteroid-centered *R-T-N* coordinate system (see discussion in Sec. II) after the last observations were made in 1986. The radial (Sun–asteroid) position uncertainty reaches a minimum when the radial velocity of the asteroid passes through zero (at aphelion and perihelion), while the transverse asteroid position uncertainty reaches a maximum at perihelion when the transverse velocity peaks. In the absence of astrometric observations beyond 1986, the radial and transverse position uncertainties grow with time, but certainly not in a linear fashion. The position uncertainty normal to the asteroid orbit plane is always very small relative to those in the two other coordinates. Figure 3 shows the general trend for the time history of ephemeris uncertainties of Earth-approaching asteroids in the absence of further observations. The absolute size of the uncertainties will change with the asteroid under study, but the trends of the uncertainties, expressed in this coordinate system, will remain qualitatively the same.

Radar-range and Doppler observations of 1986 DA were made during 18–21 April 1986 at Arecibo (Ostro *et al.* 1986a), but we have used either April 10 or April 10 and 20 as the radar observation epochs in our simulations. Table IV gives the ephemeris uncertainties of asteroid 1986 DA on 3 July 1991—a date near the middle of a several-week period when the asteroid should be optically observable.

Table IV shows that the uncertainties are insensitive to Doppler accuracy when only one Doppler observation is made, whereas there is a rather dramatic sensitivity to Doppler accuracy when two Doppler observations are made. Adding a single range measurement to two Doppler observations reduces the plane-of-sky uncertainties by a fac-

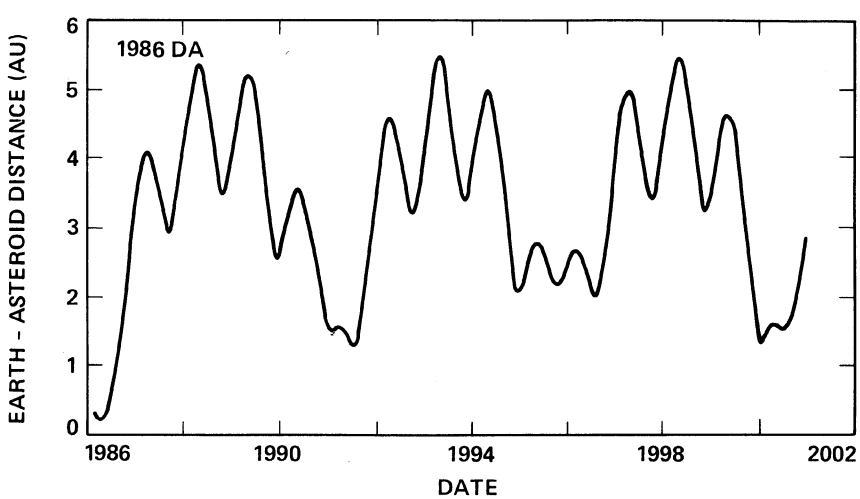

FIG. 2. 1986 DA: Asteroid's geocentric distance (AU) versus calendar date.

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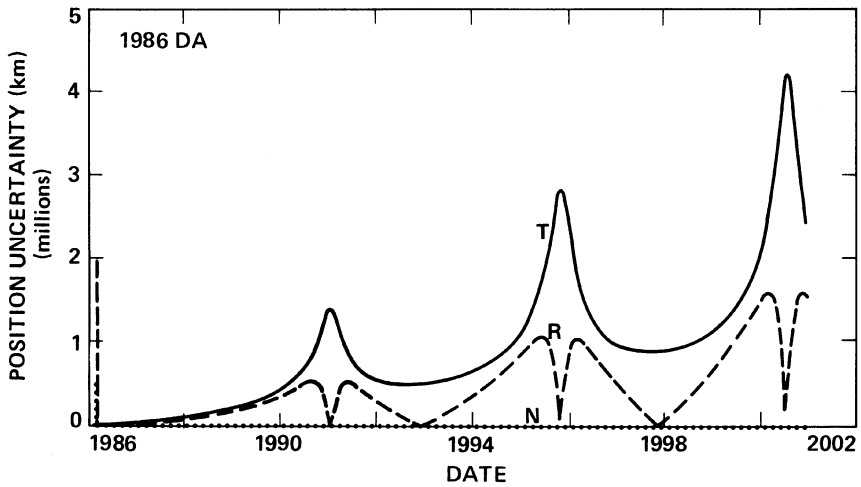

FIG. 3. 1986 DA: Asteroid's position uncertainties versus calendar date. The asteroid-centered coordinate system is defined by the Sun–asteroid unit vector (**R**), the unit vector (**N**) normal to the orbit plane, and a transverse unit vector defined as $\mathbf{T} = \mathbf{N} \times \mathbf{R}$.

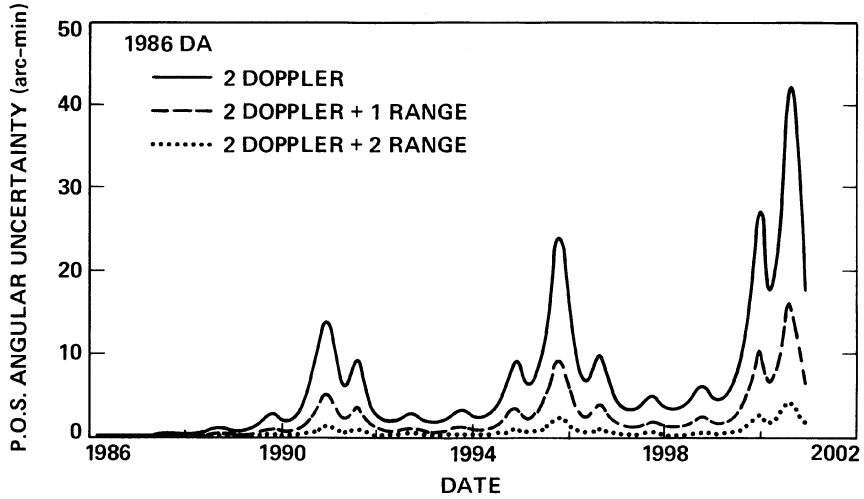

FIG. 4. 1986 DA: Plane-of-sky position uncertainties versus calendar date. Curves show sensitivity of uncertainties to number of range measurements. The range and Doppler-noise values were assumed to be 0.5 km and 30 cm/s.

TABLE IV. Plane-of-sky angular-position uncertainties for 1986 DA on 3 July 1991. Radar observations were simulated either on 10 April or on 20 April 1986, and no astrometric observations beyond 1986 were assumed.

	Plane-of-sky uncertainty (arc-min.)		Range uncertainty (units of 10^5 km)	
Doppler noise = Dataset	1 cm/s	30 cm/s	1 cm/s	30 cm/s
No radar Observations	11.1	11.1	6.50	6.50
1 Doppler obs.	10.1	10.6	5.90	6.22
2 Doppler obs.	1.47	9.01	0.852	5.26
2 Doppler obs. & 1 range obs. (range noise = 10 km)	0.43	3.40	0.249	2.01
2 Doppler obs. & 1 range obs. (range noise = 0.5 km)	0.43	3.40	0.249	2.01
2 Doppler obs. & 2 range obs. (range noise = 10 km)	0.43	0.95	0.249	0.556
2 Doppler obs. & 2 range obs. (range noise = 0.5 km)	0.42	0.91	0.247	0.532

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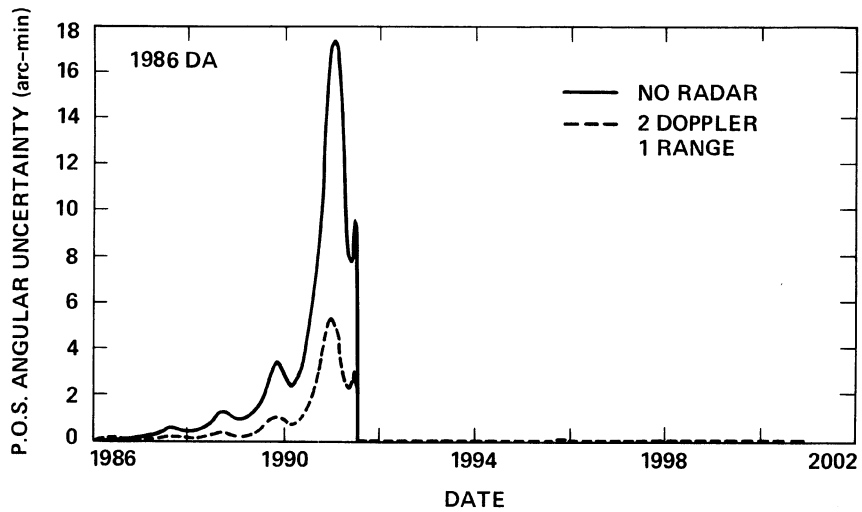

FIG. 5. 1986 DA: Plane-of-sky position uncertainties versus calendar date. Curves show dramatic improvement when 1991–1999 astrometric data are included in ephemeris predictions. The range and Doppler-noise values were assumed to be 0.5 km and 30 cm/s.

tor of 3. A single range observation is just as good as two when Doppler accuracy is high, but a second range measurement becomes very valuable when the Doppler accuracy is coarse. The sensitivity of the uncertainties to achievable values of range accuracies is negligible. Note that the uncertainties in the distance predictions are nearly proportional to the plane-of-sky uncertainties. That is, the effect of radar astrometry is to reduce the future error ellipsoid's size without significantly affecting its shape.

Figure 4 demonstrates the time dependence of the plane-of-sky position uncertainties for the three cases where there are two Doppler observations (noise = 30 cm/s) and zero, one, or two range observations (noise = 0.5 km). While the nonlinear growth of position uncertainties is evident, the percentage improvement caused by the radar observations is fairly constant with time.

As Table IV and Fig. 4 demonstrate, inclusion of 1986 radar astrometry greatly reduces the ephemeris uncertainties in the period following the radar observations. This reduction is far more dramatic (in absolute as well as relative terms) than that for Ivar because 1986 DA's optical-data history is much shorter than Ivar's. If we assume that 1986 DA is recovered in 1991, as seems likely in view of the strength of this object's actual optical and radar data sets, then the inclusion of 1991 optical observations in this object's orbit will cause the plane-of-sky position uncertainties to drop precipitously (see Fig. 5). Furthermore, inclusion of the simulated optical observations through 1999 (see Table II) keeps the uncertainties at the arcsecond level throughout the 1990s. However, it is interesting to note that during that decade the positive effect of the 1986 radar data remains evident, halving the uncertainties attainable with the optical data alone. For example, during the 1995 return, we find the maximum plane-of-sky uncertainty to be about 4″ if we exclude the radar data and about 2″ if we include it. The potential value of such an improvement to searches for stellar occultation events could be appreciable.

Recently, Gehrels (1986a) provided optical astrometric data for 1986 DA in July 1986 so that the recovery of this object in 1991 should be assured. However, these data were not included in our analysis.

c) Case Study #3: 1986 JK

This asteroid was discovered by C.S. and E.M. Shoemaker on 4 May 1986 (Shoemaker and Shoemaker 1986), and we

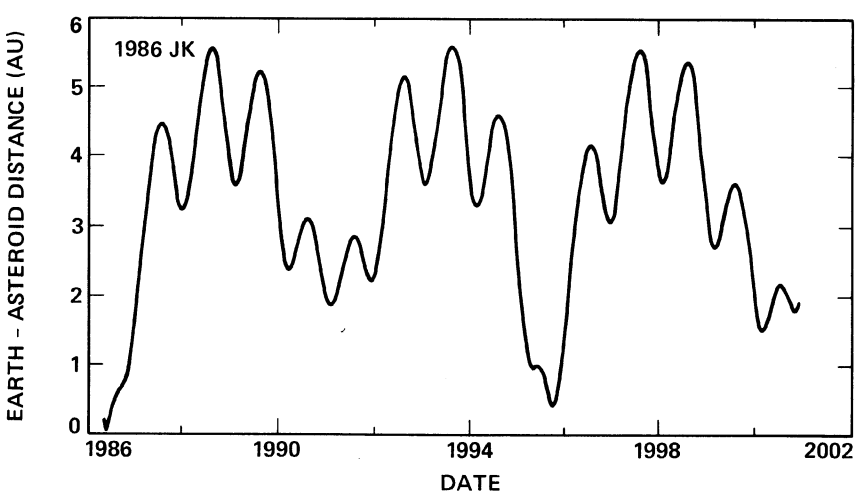

FIG. 6. 1986 JK: Asteroid's geocentric distance (AU) versus calendar date.

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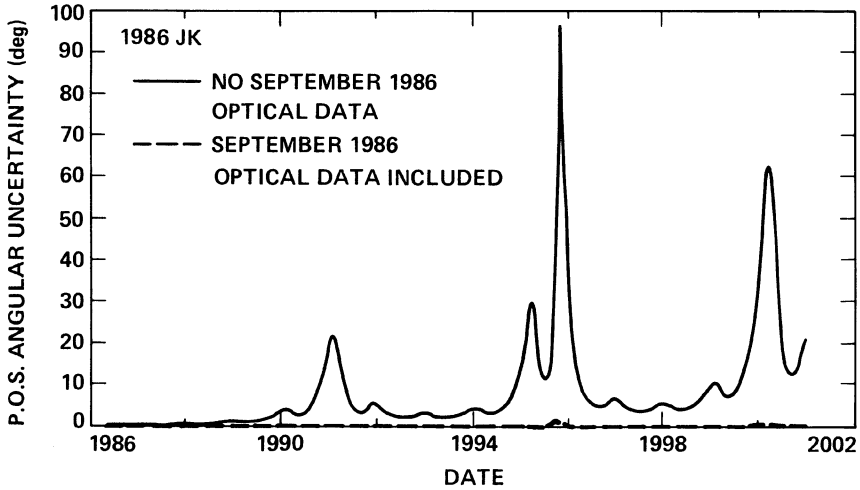

FIG. 7. 1986 JK: Plane-of-sky position uncertainties versus calendar date. Curves show position uncertainties when September 1986 optical, astrometric data are included, and when they are not.

have used 29 observations over the May 4–27 period. Simulated observations were also assumed to have been made in September 1986 and when the asteroid's computed motion suggests another Earth approach in October 1995. Figure 6 shows the Earth–asteroid distance as a function of time over the interval 1986–2001, and Fig. 7 shows the plane-of-sky position uncertainties as a function of time with and without optical astrometry in September 1986. Note the considerable benefit of the longer astrometric data interval provided by the September 1986 observations.

During the extremely close approach of this asteroid to the Earth (to within 0.029 AU) in late May 1986, successful radar Doppler observations were made at the Goldstone antenna in California (Ostro *et al.* 1986b); the actual dates of the radar measurements were used in our calculations.

If we assume that the asteroid was observed only optically during May 4–27, and that no radar or subsequent astrometric observations are made, the plane-of-sky position uncertainty on 10 October 1995 is 97° (Fig. 7). However, the addition of two Doppler observations (with noise values of 2 cm/s) on 28 May and 30 May 1986 reduces this uncertainty to 23′. That is, even without any optical observations beyond the discovery month, the radar Doppler measurements appear capable of refining our knowledge of this object's orbit so much that they ensure optical recovery in 1995. Clearly, 1986 JK demonstrates the powerful capability of radar observations to improve the ephemerides of newly discovered, near-Earth asteroids.

Recently, recovery of this object in 1995 became far more likely when optical astrometric positions of 1986 JK became available for July, August, and September 1986 (Gibson 1986; Gehrels 1986b). Table V presents results that apply if we assume that the astrometric data interval for 1986 JK extends from May 4–27 with two additional simulated observations being made in September 1986.

Similar to the analysis for 1986 DA, when two Doppler observations were taken, there is a great sensitivity of the 1986 JK plane-of-sky ephemeris uncertainties to Doppler accuracy. For this object, there is also some sensitivity to Doppler accuracy when only one Doppler observation is processed—a sensitivity that was not obvious with 1986 DA. The uncertainties are sensitive to the addition of one range

TABLE V. Plane-of-sky angular-position uncertainties for 1986 JK on 10 October 1995. Radar observations were simulated either on 30 May and/or 28 May, 1986, and no astrometric observations beyond 1986 were assumed. When one radar observation was simulated, it was taken on 28 May (Doppler) or 30 May 1986 (range).

	Plane-of-sky uncertainties (arc-minutes)		Range uncertainties (units of 10^5 km)	
Doppler noise = Dataset	2 cm/s	20 cm/s	2 cm/s	20 cm/s
No radar Observations	76.6	76.6	12.1	12.1
1 Doppler obs.	35.4	65.6	5.58	10.4
2 Doppler obs.	7.39	33.8	1.17	5.33
2 Doppler obs. & 1 range obs. (range noise = 2 km)	2.29	5.52	0.361	0.870
2 Doppler obs. & 1 range obs. (range noise = 0.2 km)	2.24	5.47	0.354	0.862
2 Doppler obs. & 2 range obs. (range noise = 2 km)	2.28	5.32	0.360	0.838
2 Doppler obs. & 2 range obs. (range noise = 0.2 km)	2.24	5.22	0.354	0.823

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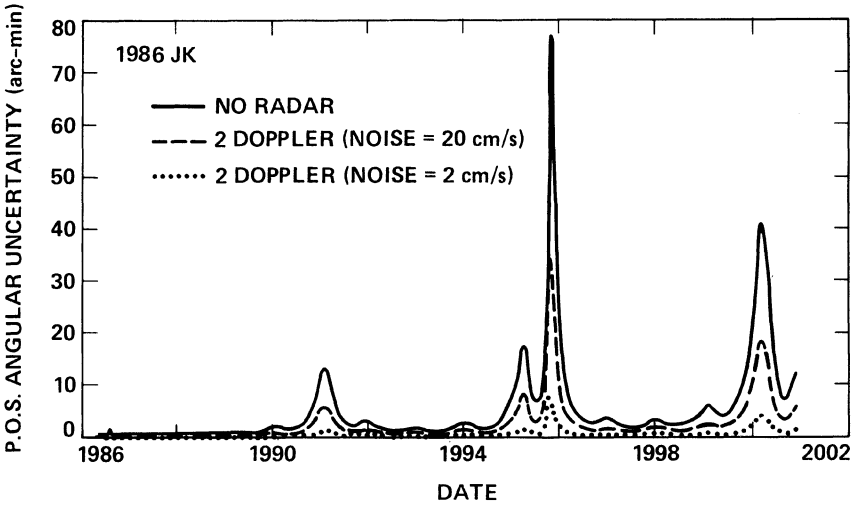

FIG. 8. 1986 JK: Plane-of-sky position uncertainties versus calendar date. Curves show sensitivity of position uncertainties to Doppler accuracy.

point but not to the accuracy of that range point, and there appears to be little improvement with the addition of a second range observation.

Figure 8 presents the time history of the position uncertainties when two Doppler observations are made at two different noise values. (Note that the curve representing no radar observations is the lower curve in Fig. 7 drawn to a different scale). The bottom curve in Fig. 8 shows the ease with which 1986 JK can be recovered in 1995 if the existing Doppler observations can be included in the orbit-determination process.

d) Case Study #4: 1982 DB

This asteroid was discovered on 28 February 1982 (Helin 1982), when it was coming out of solar conjunction, 0.1 AU from the Earth and receding in the southern sky. Had it been discovered a few weeks earlier, radar measurements would have been possible and the resulting astrometry could have ensured recovery after its discovery apparition. Unfortunately, 1982 DB exited the Arecibo and Goldstone observation windows prior to discovery. Attempts to recover this object in 1984 and 1986 were unsuccessful and it is now considered lost.

Table VI shows the effects of simulated radar astrometry, taken on 7 and 14 March 1982 upon the asteroid's ephemeris uncertainties for 27 August 1990, during the next favorable opportunity for optical recovery. Acquisition of a single, high-precision Doppler datum reduces the plane-of-sky uncertainty by more than an order of magnitude, once again demonstrating the potential of radar astrometry in securing the orbits of newly discovered near-Earth asteroids.

From Table VI and Fig. 9, we note that the plane-of-sky position uncertainties are sensitive to Doppler accuracy when one or two Doppler measurements are made. As in the other case studies, while the results are sensitive to the addition of one range point to two Doppler observations, there is little sensitivity to range accuracy and only a modest sensitivity to the addition of a second range observation.

Figure 10 displays the geocentric distance of 1982 DB as a function of time and shows that another close Earth approach is due in late 2001. In fact, a separate numerical integration confirms that the computed motion of the asteroid will bring it to within 0.028 AU of the Earth on 31 December 2001—the first New Year's Eve in the next millennium. Using just the 1982 optical data (i.e., the only data actually available now), the plane-of-sky and range uncertainties during the close Earth approach are 55° and 9.4×10^5 km, or

TABLE VI. Plane-of-sky angular-position uncertainties for 1982 DB on 27 August 1990. No astrometric observations beyond 1982 are assumed. Radar observations were simulated on 14 March 1982 and/or 7 March 1982.

	Plane-of-sky uncertainties (arc-min.)		Range uncertainties (units of 10^5 km)	
Doppler accuracy =	2 cm/s	20 cm/s	2 cm/s	20 cm/s
Dataset				
No radar Observations	40.2	40.2	2.06	2.06
1 Doppler obs.	3.5	6.7	0.173	0.340
2 Doppler obs.	2.3	4.9	0.114	0.248
2 Doppler obs. & 1 range obs. (range noise = 30 km)	0.69	2.7	0.0404	0.143
2 Doppler obs. & 1 range obs. (range noise = 0.3 km)	0.69	2.7	0.0402	0.143
2 Doppler obs. & 2 range obs. (range noise = 30 km)	0.68	1.34	0.0400	0.0728
2 Doppler obs. & 2 range obs. (range noise = 0.3 km)	0.63	0.64	0.0374	0.0378

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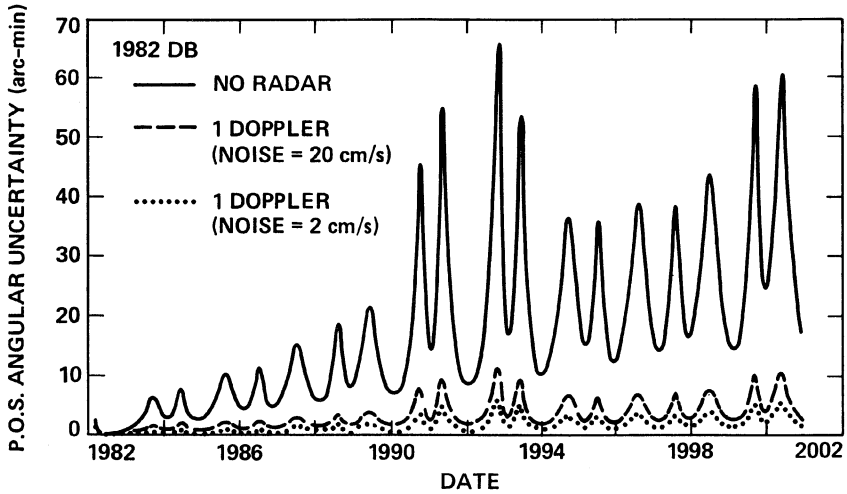

FIG. 9. 1982 DB: Plane-of-sky position uncertainties versus calendar date. Curves show sensitivity of position uncertainties to Doppler accuracy when only one radar observation is made.

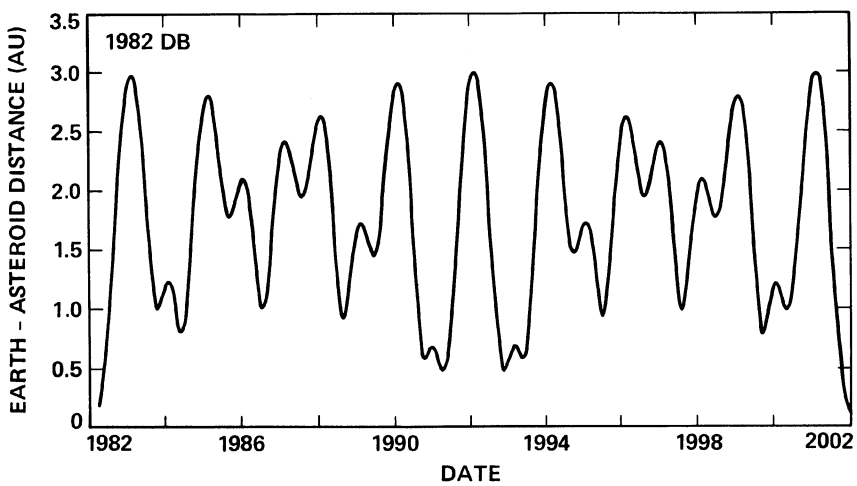

FIG. 10. 1982 DB: Asteroid's geocentric distance (AU) versus calendar date.

0.0063 AU. Hence, if this asteroid is not recovered in 1990, its location on the sky during the 2001 close approach will be essentially unknown and the uncertainty in the closest-approach distance will be about 25% of the distance prediction itself. Because of the reasons outlined in Sec. II, even this large "formal" uncertainty is likely to be much smaller than the "true" uncertainty. Of course, optical recovery in 1990–1992 could reduce these uncertainties by three orders of magnitude.

IV. CONCLUSIONS

For first apparition, close Earth-approaching asteroids, radar observations provide an extremely powerful technique for improving the accuracy of their orbits and future ephemerides. For newly discovered asteroids with short observation intervals, optical astrometric data alone are often not sufficient to prevent their being lost. Successful radar observations could secure such an object's orbit and ensure its recovery during a subsequent Earth approach.

Once asteroids are recovered optically on their second (and subsequent) apparitions, their orbits are vastly improved and generally secure, with or without radar observations. However, even for asteroids with very secure orbits, the several-fold improvement in the accuracy of future ephemerides from radar observations would be potentially valuable for occultation predictions, dynamical studies, or if the asteroid should become a target for a future flyby or rendezvous space mission. The percentage improvement in future ephemerides due to the addition of radar data is nearly constant for at least a few decades.

From an ephemeris-improvement standpoint, the optimal time to make Doppler astrometric observations with a given level of accuracy is when the Earth–asteroid radial velocity is largest—unfortunately not at the closest-approach point, where echoes are strongest. When only one Doppler observation is made, the sensitivity of the ephemeris uncertainties to the Doppler accuracy is not as impressive as when two Doppler observations are made. Future ephemeris uncertainties are very sensitive to the addition of one range obser-

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vation, but there is little sensitivity to range accuracy and only a modest sensitivity to the addition of a second range observation.

Our results, which underscore the potential of radar astrometry for improving the future ephemerides of Earth-approaching asteroids, also apply to Earth-approaching, short-period comets. However, various additional systematic sources of error in making the optical measurements and in modeling nongravitational phenomena (Yeomans *et al.* 1983) would render the ratio of actual error to calculated error much larger for comets than for asteroids.

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